Cathodic Protection Numeric Method Summary Report

BOWEN XU

Introduction

The project is object to build and solve a mathematical model using a numerical technique by Boundary Element Method (BEM) for corrosion analysis. At the end, the project is not completed successfully. Reasons are discussed and possible future improvement is outlined.

To investigate what went wrong during the project, firstly, we have to know a certain amount of physics and chemistry concepts that underlie the mathematical model.

Chemistry Background:

External corrosion on pipelines can be mitigated by coating the pipe with a high-resistance film. Cathodic protection (technology used to prevent or reduce the metal corrosion by making the corroding metal a cathode by connecting to a sacrificial anode) is used to protect portions of the pipe where coating holidays exists. The purpose of building a mathematical model is to investigate the effect of discrete coating holidays of various sizes on the performance of the CP system.

Mathematical Model

The model is built upon the interior Laplace's equation, which in 3-D setting, can be written as $\nabla^2 \Phi = \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0$ (1) where Φ is the electrical potential. There are a number of assumptions and boundary conditions that need to be considered. It is listed below:

- The concentrations (parameter κ) are uniform
- $\kappa \nabla \Phi = i$ where i stands for current

Because in this case, the domain is infinite, we try to use BEM to solve the problem. The main advantage of using BEM over FEM (probably more popular and commonly known) is that it offers much more accurate field calculations especially in 3D setting

The first stage of the BEM process is to rewrite the partial differential equation (1) as an integral, therefore an influence function called Green function is introduced. Green's functions are essential to the Boundary Element Method. There are two different types of method: direct and indirect. The direct method requires the use of Green's second theorem

$\phi \nabla^2 f - f \nabla^2 \Phi = \nabla(\Phi \nabla f - f \nabla \Phi)$

and Green's function of Laplace equation states that $\nabla G(x, x0) + \delta_3(x - x0) = 0$ where x and x0 are 3-dimensional vector field points and $\delta_3(x - x0)$is a 3-D delta function.

In free space setting, G(x,x0)=$\frac{1}{4\pi r}$

Due to the lack of understanding for the chemical mechanism, this step (integral representation of Green’s function) can not proceeded accordingly. Neither had clear understand the purpose of doing this and how does it help us to solve the problem. And the actual integral solving and meshing part of program requires the use of Matlab, in which the FORTRAN code was provided. Fail to understand the method, as well as the code jeopardised the progress.

During the procedure, there are also a number of unfamiliar concepts were identified, such as the Laplace Integral Operator. It was not prepared for this challenge. The lack of understanding in the entirety of project, lack of guidance and desire to seek such guidance are all contributing to this fruitless effort.

Reference:

Harwood, Adrian R.G. and Iain D.J. Dupère. "Calculation Of Acoustic Green’S Functions Using BEM And Dirichlet-To-Neumann-Type Boundary Conditions". *Applied Mathematical Modelling* 39.14 (2015): 4134-4150. Web.

Pozrikidis, C. *A Practical Guide To Boundary Element Methods With The Software Library BEMLIB*. 1st ed. Hoboken: CRC Press, 2002. Print.